DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2012

In

MATHEMATICS

April 14th 2012

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables are available for use

Mr John Turner
Dr Martin Rogers

Ms. Marisa Llorens-Salvador

P.T.O.

1 (a) Express x^2+x+11 in the form $a(x+b)^2+c$

(25)

(b) Given that x+3 is a factor of ax^3-x^2+4x-6, find a.

(25)

(c) Solve the simultaneous equations:

$$x - y = 1$$
$$2xy + y = 3$$

(25)

(d) It takes 10 days to lay high voltage cable between the first two towers of a line. Each additional tower takes 2 days longer than the one before. Set up the arithmetic progression for this problem and find its nth term and the total time needed for n towers. What length (in number of towers) can be built in 30 days?

(25)

2. (a) Find values of the first derivatives of the following at the given points

(i) $y= (3x^2+4x)^5$ at x=1.5 (9)

(ii) $f(x) = \frac{x^2+x-2}{5x-3}$ at x= 3 (9)

(iii) $g(x) = 5x + \sqrt{x^2 - 2}$ at x=2 (7)

(b) T is given by $T = 25n + n^2$. Find the value of n for which T is a minimum and show that this value gives the minimum, not the maximum value of T.

(25)

(c) The amount of fuel, W litres, used by an aircraft flying a certain distance at a speed v (km/ min) is given by $W = 5v^2 + \frac{125}{v}$

(i) Find the rate of change of W with respect to v when v=4km/min.

(ii) Find the smallest number of litres required for the distance and the most economical cruising speed.

(25)

(d) Given the function $y = \frac{x-kx^3}{x^2}; x \neq 0$ and that $\frac{dy}{dx} = 0 \; for \; x = \frac{-2}{3}$, find the value of k. (25)

P.T.O.

3. (a) A sum of €1000 invested on January 1st 2012 attracting an annual interest rate of 5% per annum. By what factor does this sum increase each year and how much will it be worth on January 1st 2017?.

(25)

(b) Given that (x-2) is a factor of $2x^3+3x^2-18x+8$, find all 3 roots.

(25)

(c) Solve for x:

(i) $2log_2(x+2) - log_2(x^2+2x+1) = 4$ (10)

(ii) $\ln\frac{4x+1}{x-2} = 2.14$ (15)

(d) A certain strain of bacteria that is growing on your kitchen counter doubles every 5 minutes following:

$$y = y_0 e^{kt}$$

Assuming that you start with only one bacterium, how many bacteria could be present at the end of 96 minutes?

(25)

4. (a) Find a and b if $a(2+3i)+b(1-i)-5=0$

(20)

(b) Let z=(1+i). Show z is a solution of both

(i) $z^2-2z+2=0$ (8)

(ii) $z^3+z^2-4z+6=0$ (12)

(c) Express $\frac{(1+2i)^2}{(2-i)}$ in the form a +ib

(20)

(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: 3i, 2+i , 3-i, 1+3i .

(20)

(e) Express $\frac{-2}{(-1+i)}$ in both a+ib and polar forms and find value of $\left(\frac{-2}{(-1+i)}\right)^9$

(20)

P.T.O.

5. (a) Express sin 3x -sin x as a product and hence find the values of x in the range $0 \leq x \leq 2\pi$ for which (sin 3x + cos 2x -sin x)=0

(25)

(b) Solve the equation $\tan(2x) = -1$ Give answer to one decimal place.

(25)

(c) From a window 8m above horizontal ground the angle of elevation of the top of a higher building across the road is 20 º and, from the same window, the angle of depression of the foot of the same building is 30º. Find the width of the road and the height of the building.

(25)

(d)

(i) An engineer has to make a triangular component from a metal sheet. The triangle is ABC and the length AB is 12 cm, the length BC is 9 cm while the included angle ABC is 115°. What are the other angles and the length of AC?

(13)

(ii) Sketch the graph for cos(x) and cos(2x).

(12)

6 (a) Show the circles $x^2+y^2-10x-6y+30=0$ and $x^2+y^2+6x-6y-18=0$ touch each other and find the point of contact.

(25)

(b) Find the equation of the circle with centre (-2,4) which touches the x axis.

(25)

(c) Find the equation of the line that passes through the point of intersection of the lines 2x-y-2=0 and x+y-10=0 and is perpendicular to the line 5y+3x=8.

(25)

(d) Solve for x: $x + 2 = \sqrt{4x + 7}$

(25)

P.T.O.

7 (a) Evaluate 3 of the following integrals: (25 each)

(i) $\int x^2 \cos x \, dx$

(ii) $\int_1^3 3t(t^2+2)^8 \, dt$

(iii) $\int_0^{\pi/2} \sin 4x \cos 3x \, dx$

(iv) $\int \frac{2x+1}{(x-1)(x+2)} dx$

(b) Sketch the curve $y=2x^2+2$ between x=-2 and x=3. Calculate the area enclosed by the curve, the x axis and the lines x=-1 and x=3.

(25)

8. (a) Find a and b if A is the inverse of B:

A= $\begin{pmatrix} a-1 & 2 & 1 \\ 1 & a & a \\ 1 & 1 & 1 \end{pmatrix}$ B= $\begin{pmatrix} 0 & -b & 2 \\ b & 0 & -b \\ -b & 1 & 0 \end{pmatrix}$

(20)

(b) and hence solve the set of equations using B

$$\begin{array}{llll} (a-1)x & +2y & +z & =1 \\ x & +ay & +az & =2 \\ x & +y & +z & =3 \end{array}$$

(20)

(c) Calculate the inverse of $D = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

(20)

(d) Use Gaussian elimination to solve the following system of simultaneous equations:

$$x + y = 4$$

$$3x - y = 2$$

(20)

(e) If $C = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Find k so that $C^2 + kC + I = \begin{pmatrix} 4 & 4 & 4 \\ 7 & 7 & 6 \\ 5 & 5 & 6 \end{pmatrix}$

(20)