

**DUBLIN INSTITUTE OF TECHNOLOGY**

**First Year Engineering Entrance Examination 2013**

**In**

**MATHEMATICS**

**August 2013**

**Attempt any 6 of the following 8 QUESTIONS**

**Time Allowed: 3 hours**

**Each question has 100 marks**

**All question carry equal marks**

**Maths Tables are available for use**

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**P.T.O.**

- 1 (a) The equivalent capacitance  $C$  of a network of 3 capacitors connected in series is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Express the right hand side of this equation as a single fraction and hence obtain an expression for  $C$ .

(25)

- (b) Find the roots of  $x^2 + 3x - 5$ .

(25)

- (c) Solve the simultaneous equations:

$$\begin{aligned} 3x + 4y &= 18 \\ 4x + 5y &= 23 \end{aligned}$$

(25)

- (d) Find  $a$  and  $b$  given that  $(x+2)(x-3)$  are factors of

$$f(x) = 2x^3 + ax^2 + bx + 60$$

(25)

2. (a) Find values of the first derivatives of the following at the given points:

(25 each)

(i)  $f(x) = \frac{e^{-t}}{t^2+2}$  at  $t=0$

(ii)  $g(x) = \sin(x^2 + 3x)$  at  $x=0$

- (b) Given the function  $y = x^3 - 4x^2 + 10$ . Find the two turning points and specify if they are maximum or minimum points.

(25)

- (c) Given the function  $y = \frac{3x^2+kx-1}{x+1}$ ;  $x \neq 0$  and that  $\frac{dy}{dx} = 0$  for

$x = 0.154$ , find the value of  $k$ .

(25)

3. (a) Find the 10<sup>th</sup> term and sum of the first ten terms of the series 1+2+4+8...
- (25)
- (b) Show  $x=2$  is a root of the cubic  $x^3 + 13x^2 + 6x - 72$ . Find other roots.
- (25)
- (c) Solve for  $x$ :
- (i)  $2 \log_{10} (x+1) - 2 \log_{10} (x-1) = 4$  (10)
- (ii)  $\ln x + \ln(x + 1) = 1$  (15)
- (d) The amount of chemical in a reaction after  $t$  minutes is given by
- $M = 25 e^{-t^2}$  in grams.
- How much material is left after 1 minute and estimate how long it will take for the amount of the chemical to reduce to 2 grams?
- (25)
4. (a) Given  $z = -1-2i$  and  $h = 2-i$ . Calculate  $z + h$ ,  $z - h$ ,  $z \cdot h$  and  $\frac{z}{h}$ .
- (25)
- (b) Show that  $z = 3-i$  is a root of  $z^3 - 2z^2 - 2 + 14i = 0$ .
- (25)
- (c) Express  $(-3-6i)$  in polar form and calculate  $(-3-6i)^4$ .
- (25)
- (d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form:  $2+i$ ,  $-1+3i$ ,  $-2-2i$ ,  $-6i$ ,  $2-3i$ .
- (25)

5. (a) The angle of elevation of a tower from a point on the ground is  $30^\circ$ . The angle of elevation increases by  $4^\circ$  when measured from a second point also on the ground, but 22 m nearer the tower than the first point. Find the height of the tower and the distance from the top of the tower to the first and second points. (25)

(b)

- i) Given the triangle ABC with  $A=62^\circ$ ,  $BC=8$  m and  $AB=7$  m. Find all the other sides and angles. (13)

- ii) Given the triangle DEF with  $E=78^\circ$ ,  $Fd=39$  m and  $D=37^\circ$ . Find all the other sides and angles. (12)

- (c) Sketch the graph for  $2\cos(x)$  and  $\cos(2x)$  between 0 and  $2\pi$ . (25)

- (d) Solve the equation  $\cos(2x) = -0.78$  for  $0 \leq x \leq 360^\circ$  (25)

- 6 (a) The points A(3,4) and B(9,12) are two ends of a diameter of a circle. Find the equation of the circle. (25)

- (b) Show that the circle  $(x - 2)^2 + (y - 3)^2 = 4$  touches the y axis and find the point where it touches it. (25)

- (c) Find the equation of the line that passes through the point of intersection of the lines  $x+2y-3=0$  and  $x-y-2=0$  and is parallel to the line  $y+3x=4$ . (25)

- (d) Solve for x:  $(2x + 3)^2 = (3x - 6)^2$  (25)

**P.T.O.**

7 (a) Evaluate 3 of the following integrals: (25 each)

(i)  $\int 3 + 4x - \frac{2x^2}{x^4} dx$

(ii)  $\int (2x + 1) \frac{1}{x^2 + x + 25} dx$

(iii)  $\int \frac{11-3x}{(x-1)(x+3)} dx$

(iv)  $\int_0^1 e^{2x} 3x dx$

(b) Find the area under the curve  $y = x^3 - 4x^2 + 10$  between the values  $x=-1$  and  $x=3$ . (25)

8. (a) Find the inverse of A and prove that the result is correct.

$$A = \begin{pmatrix} 3 & -2 \\ 1 & 2 \end{pmatrix} \quad (25)$$

(b) Given the following matrices calculate:  $C*B$ ,  $B*C$ ,  $B*D$  and  $D*B$ , if the operations are feasible. Explain your reasoning.

$$B = \begin{pmatrix} 3 & 4 & 4 \\ 1 & 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 3 \\ 2 & -3 \\ 1 & 3 \end{pmatrix} \quad D = \begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix} \quad (25)$$

(c) Use Gaussian elimination to solve the following system of simultaneous equations:

$$\begin{aligned} x - y &= 2 \\ 2x + y &= 5 \end{aligned} \quad (25)$$

(d) Given  $G = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ . Find k:

$$G^2 + G + \begin{pmatrix} k & k \\ 1 & k \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 2 & 2 \end{pmatrix} \quad (25)$$