DIT
SPECIAL MATHS PAPER
APRIL 2013

SAMPLE SOLUTIONS

Q1. a) $2(x+1)^2+10$

b) x3+ax2-5x-6 / x+3 = x2 + (a-3)x+(-3a+4) remainder -9a+18 as x+3 is a factor -9a +18=0 and a = 2

c) x-y=4 x=4+y
4+y+3(4+y)y=20 ➔ 3y2+13y-16=0
y=1 and y= -5.33
therefore x= 5 and x= -1.33

d)
a=35
d=4

$$S_{100} = \frac{1}{2}\, 100(2 * 35 + (99) * 4) = 23300$$

Q2. a) i)y'= cos(3x+50)*3 at x= $\pi/2$ y'= 3cos(3π/2 +50)=-0.78712
ii) f'(x)=(4x+3)sin(x) + cos(x) (2x^2+3x-5) at x=0 f'(x) = 0-5=-5
iii) g'(x)= ½ $(x^2 + 2x - 1)^{-\frac{1}{2}}\,(2x + 2)$ at x=2 $g'(2) = \frac{1}{2}\frac{6}{(7)^{0.5}} = 1.133893$

b) $y' = 3x^2 + 4x - 1$
$3x^2 + 4x - 1 = 0$
x=0.215 and x=-1.548
y''=6x+4
for x=0.215 y''= + >0 Min
for x= -1.548 y''= - <0 Max

c) $x' = 200 - \frac{25}{2}2t = 200 - 25t$
when t=3 x'= 200-25*3= 125 m

ii)200-25t=0
t= 200/25= 8
x''= -25 <0 Max
x(t=8) = 200*8-25/2*64 = 800

d)
y'= 1/2(2x^2+kx-1)^(-1/2) (4x+k)
0=1/2 (2(-1/2)^2+k(-1/2)-1)^(-1/2) (4(-1/2+k)
0=-2+k
k=2

Q3. a=2
r=0.8

$$S\infty = 2 * \frac{2}{1-0.8} = 20\ m$$

b)
2^3-2*2^2-2+2 = 8-8-2+2 =0 therefore x=2 is a root of the cubic function.

$$(x^3 - 2x^2 - x + 2)/(x-2) = x^2 - 1$$

x=+1 and x=-1 are the other roots

c) 3 log (2x-5)=1- log (10) =0
log (2x-5)=0
2x-5=10^0 = 1
2x=1+5
x=6/2=3

ii) $\ln \frac{x}{x-3} = 0.2$

$\frac{x}{x-3} = e^{0.2} = 1.221403$

x= 1.22x- 3*1.22 = 1.22x- 3.66
0.22 x = 3.66
x= 3.66/0.22 =16.63

d) $M(t=1) = 25\ e^{-0.2} = 20.468$
$12 = 25\ e^{-0.2t}$ 12/25= $e^{-0.2t}$ ln 0.48= -0.2t
t= 3.669

Q4. 3a-b-5=0
-4a-2b=0

a=1
b=-2

b) z1*z2= (1-2i)(-1+2i)= -1+2i+2i-4i^2 = -1+4i +4 = 3+4i

z1/z2 = -1+2i-2i+4i^2/5 =-1-4/5i=-5/5=-1

c) (2+i)^3+(2+i)^2-15(2+i)+25=0
8+12i+6(-1)+(-i)+4+4i-1-30-15i+25=0
8-6+4-1-30+25=0
12i-i+4i-15i=0
.
d)1+2i/2-3i= -0.30769 + 0.53846 i = 0.62 < 2.0899 rad=119.7449 degrees
z^4= 0.147929 <118.9795 degrees

e)

1+2i	2.236068	63.43495
2-3i	3.605551	303.69
3-3i	4.242641	315
1+3i	3.162278	71.56505
-5-i	5.09902	191.31
6i	6	90
-2+4i	4.472136	116.5651

Q5.

a)

x=6/tan15=22.39 m width of the road

*y= 22.39*tan(30)= 12.92 height building= 12.92+6=18.92 m*

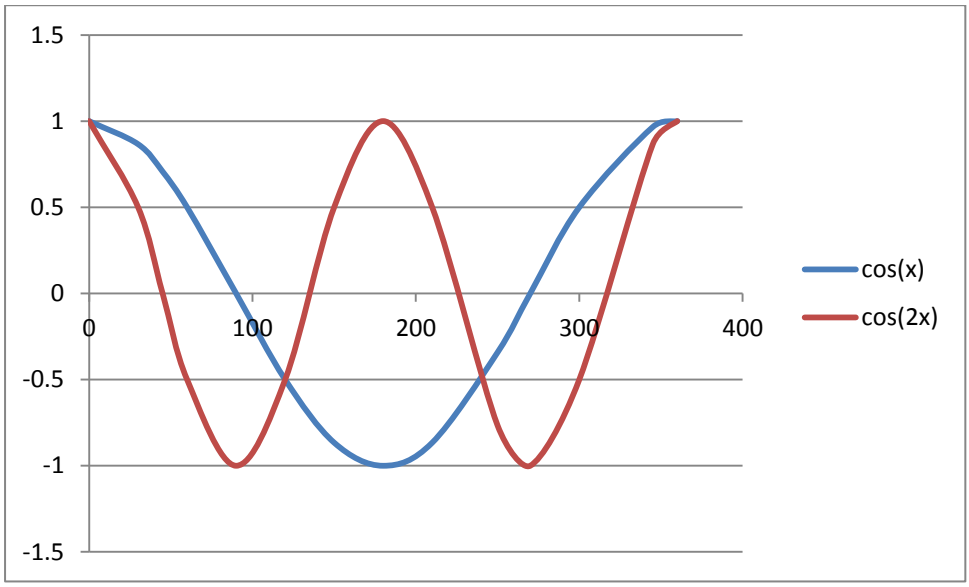
b)

sine rule

*sin c=17.92*sin(78)/22.31=0.785675 ➔ c= 51.78 degrees*

A= 180 – 78 – 51.78 =50.21 degrees

*a=22.31*sin(50.21)/sin(78)=17.41*



c)2x= cos-1(-0.5)=120➔ x=60

2x=240➔ x=120

2x=360+120= 480 ➔ x=240 not in the interval

2x= 360+240= 600 ➔ x=300 not in interval

d)

$\cos(y-\pi)+\sin\left(y+\frac{\pi}{2}\right)=0$ using cos(a-b)= cos(a) cos(b) + sin(a) sin(b) and sin(a+b) = sin(a) cos(b) + sin(b) cos(a)

cos(y)cos π +sin y sin π+ sin y cos π/2 + sin π/2cos y = 0
-cos(y) + 0+0+cos(y) = 0

Q6.

(1,2) (4,6) ➔ $d=\sqrt{(4-1)^2+(6-2)^2}=5$ same as radius
(1,2) (5,-1) ➔ $d=\sqrt{(5-1)^2+(-1-2)^2}=5$ same as radius
(1,2) (-3,5)➔ $d=\sqrt{(-3-1)^2+(5-2)^2}=5$ same as radius
(1,2) (-2, -2)➔ $d=\sqrt{(-2-1)^2+(-2-2)^2}=5$ same as radius

b)2,6 CENTRE radius 2

$$(x-2)^2+(y-6)^2=2^2$$

c)intersection x=1.166 and y=0.333
m=-5 mperpendicular= 1/5
y-0.333=1/5(x-1.166)

d) $3x+1=\sqrt{2x+10}$
9x^2+4x-9=0
x=0.8 and x=-1.24

Q7.

(i) $\int_1^2 \frac{4x+2}{(x^2+x-9)^3}dx=2\left[\frac{u^{-2}}{-2}\right]=-(-3)^{-2}--(-7)^{-2}=-\frac{1}{9}+\frac{1}{49}$

(ii) $\int_0^{\pi/2} cos^2(x)\,dx=\left[\frac{x}{2}+\frac{1}{4}\sin(2x)\right]=\frac{\pi}{4}+\frac{1}{4}sin\pi-0-0=\frac{\pi}{4}$

(iii) $\int\frac{2x-1}{(x+2)(x-3)}dx=\ln(x+2)+\ln(x-3)+c$

(iv) $\int x\,sin\,x\,dx=-x\cos x+\sin x+c$

integration by parts

(b) Find the area under the curve y = 4 x ^{3} + 3 x -1 between the values x=1 and x=4.

(25)

$\int_1^4 4x^3+3x-1\,dx=\left[x^4+3\frac{x^2}{2}-x\right]=4^4+24-4-1-\frac{3}{2}+1$ =274.5

Q8.

a)

$$AB=\begin{pmatrix}k&-2&k\\-2&5k&-4\\k&-4&6\end{pmatrix}\begin{pmatrix}14&8k&3\\8k&5&2\\3&2&k\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

Using the first row first column:
14k-16k+3k=1
k=1

b)
$B^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ and x= 11 and b=-14

c) 2 1 8 =➔ ½ r1 ➔ 1 0.5 4 ➔ -3r1+r2 ➔ 1 0.5 4
3 2 5 3 2 5 0 0.5 -7
y=-14 and x= 4+7=11

d)a*B=

18	11
34	23

B*A=

11	5	8
10	6	8
30	18	24

A*C is not feasible