

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2012

In

MATHEMATICS

August 2012

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables are available for use

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P.T.O.

1 (a) Express $2x^2+16x+39$ in the form $a(x+b)^2 + c$ (25)

(b) Given that $x-1$ is a factor of $x^3 + a x + 6$, find a . (25)

(c) Solve the simultaneous equations:

$$\begin{aligned}x + y &= 1 \\2xy - y &= -10\end{aligned}$$

(25)

(d) An engineering firm needs to replace an item of machinery and is building up a fund of €5 million to do so. The proposal is to make 12 equal deposits over a three year period to an account which earns 4% interest compounded quarterly. What is the amount of each deposit? (25)

2. (a) Find values of the first derivatives of the following at the given points

(i) $y = \cos(x^2 - x - 2)$ at $x = \pi$ (9)

(ii) $f(x) = \frac{-x+3}{2x-1}$ at $x = 3$ (9)

(iii) $g(x) = 2x^5 - (x^2 + 2x - 1)^2$ at $x = 5$ (7)

(b) P is given by $P = 2n^3 - 5n$. Find the value of n for which P is a maximum and show that this value gives the maximum, not the minimum value of P . (25)

(c) The luminous intensity, I candelas of a lamp is given by $I = 6 \cdot 10^{-4} \cdot V^2$, where V is the voltage.

(i) Find the rate of change of luminous intensity with voltage when $V = 200$ volts.

(ii) Find the voltage at which the light is increasing at a rate of 0.3 candelas per volt. (25)

(d) Given the function $y = \frac{kx^2+x}{x^3}$; $x \neq 0$ and that $\frac{dy}{dx} = 0$ when $x = -2/3$, find the value of k . (25)

3. (a) A total of €11700 is to be shared amongst three people. The common ratio is r and the smallest share is €1200. Show that r satisfies the equation $4r^2 + 4r - 35 = 0$ and hence find the largest share.

(25)

(b) Show $x=2$ is a root of the cubic $x^3 - 7x + 6$. Find other roots.

(25)

(c) Solve for x :

(i) $2\log_2(2x + 3) - \log_2(x - 1) = 8$ (10)

(ii) $\ln \frac{x}{2x-1} = 4.1$ (15)

(d) In a chemical reaction the amount of starting material in cc left after t hours is given by $C = 75e^{-0.32t}$. How much material is left after 3.5 hours and estimate how long it will take for C to fall to 52 cc.

(25)

4. (a) Find a and b if $a(1+3i) - b(1-2i) - 5 = 0$

(20)

(b) If $z_1 = 2 - i$ and $z_2 = 2 + 4i$ find $z_1 * z_2$ and $\frac{z_1}{z_2}$

(20)

(c) Show that $z = 1 + i$ is a solution of $z^3 + z^2 - 4z + 6 = 0$

(20)

(d) Express $\frac{5}{2+i} - 1$ in both $a+ib$ and polar forms and find value of z^6

(20)

(e) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: $i, 1+2i, 2-i, -i, -1+3i, -2-3i$.

(20)

5. (a) Express $\sin 3x - \sin x$ as a product and hence find the values of x in the range $0 \leq x \leq 2\pi$ for which $(\sin 3x + \cos 2x - \sin x) = 0$ (25)

(b) Solve the equation $\sin(3x) = 1$ for $0 \leq x \leq 180$ (25)

(c) From a window 10m above horizontal ground the angle of elevation of the top of a higher building across the road is 52° and, from the same window, the angle of depression of the foot of the same building is 36° . Find the width of the road and the height of the building. (25)

(d)

(i) An engineer has to make a triangular component from a steel sheet. The triangle is ABC and the length AB is 14 cm, the length BC is 10 cm and the included angle is 44° . What are the values for the other 2 angles and the other side? (13)

(ii) Sketch the graph for $\sin(x)$ and $2\sin(x)$. (12)

6 (a) Show the circles $x^2 + y^2 - 6x - 4y + 12 = 0$ and $x^2 + y^2 - 4y = 0$ touch each other and find the point of contact. (25)

(b) Find the equation of the circle with centre (1,-4) which touches the x axis. (25)

(c) Find the equation of the line that passes through the point of intersection of the lines $x + y - 1 = 0$ and $2x - 4y - 10 = 0$ and is perpendicular to the line $y + 2x = 1$. (25)

(d) Solve for x : $x - 1 = \sqrt{2x + 2}$ (25)

7 (a) Evaluate 3 of the following integrals: (25 each)

(i) $\int x \sin x \, dx$

(ii) $\int_1^4 n^4 + n^2 + \frac{c}{n} \, dn$ Being C a constant.

(iii) $\int_0^{\pi/2} \sin 2x \cos 3x \, dx$

(iv) $\int_3^4 \frac{x+6}{(x-1)(x+2)} \, dx$

(b) Sketch the curve $y=2x^3-2x$ between $x=-1$ and $x=4$. Calculate the area enclosed by the curve, the x axis and the lines $x=-2$ and $x=3$.

(25)

8. (a) Find a and b if A is the inverse of B:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & a & 1 \\ a & 1 & a \end{pmatrix} \quad B = \begin{pmatrix} -0.533 & b & 0.0666 \\ 0.6 & 0 & -0.2 \\ b & -b & b \end{pmatrix} \quad (20)$$

(b) And hence solve the set of equations using B

$$\begin{array}{rcl} x & +2y & +1z = 1 \\ 4x & +ay & +z = 3 \\ ax & +y & +az = 5 \end{array} \quad (20)$$

(c) Calculate the inverse of $P = \begin{pmatrix} 4 & 8 \\ 12 & 7 \end{pmatrix}$ and calculate b if $P^{-1} \begin{pmatrix} b \\ b \end{pmatrix} = \begin{pmatrix} 0.073 \\ 0.588 \end{pmatrix}$ (20)

(d) Use Gaussian elimination to solve the following system of simultaneous equations:

$$\begin{array}{l} 2x + y = 7 \\ 3x - 2y = 9 \end{array} \quad (20)$$

(e) Given the following matrices indicate if the following operations are feasible and give the dimensions of the result matrix for each of them: $A*B$, $B*A$, $A*C$, C^{-1} . Explain your reasoning.

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 5 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 23 & 4 \\ 2 & 2 \\ 6 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} \quad (20)$$