DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2011

In

MATHEMATICS

August 24^{th} 2011

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables are available for use

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1. (a) Express $2x^2+12x+13$ in the form $a(x+b)^2+c$. (25)

(b) Given that x=3and x=-2 are both roots of the equation $3x^3+ax^2-17x+b=0$, find the values of a and b. (25)

(c) Solve the simultaneous equations: (25)

$2x-y=5$
$xy+x^2=2$

(d) A sum of € 6300 is to be shared among 3 people and the amounts are the first three terms of a geometric series. The smallest share is €900. Find value of the largest share. (25)

2. (a) Find values of the first derivatives of the following at the given points:

(i) At x=2 for $y = 3x - \frac{1}{\sqrt{x}} + \frac{1}{x}$ (20)

(ii) At x=4 for $y = \frac{x+7}{\sqrt{x}}$ (20)

(b) A car rental agency has 24 identical cars. The owner finds that she can rent all the cars if she sets the price at €10 a day. However, each time she decides to increase the price by €1, one of the cars is not rented out.
Use differentiation to find what should be charged in order to bring in the maximum revenue. (20)

(c) The speed of an object is given by $v=96+40t-4t^2$ where t is in seconds and v in metres per sec. (20)

(i) Find both times at which this speed is 96 m/sec
(ii) What is the acceleration at a time t=2.5 secs
(iii) at what value of t will the acceleration become negative

(d) Given the function $y=\frac{4+kx-2x^2}{2x^3}; x \neq 0$ and that $\frac{dy}{dx}=0$ for x=-2, find the value of k. (20)

3. (a) A sum of €10000 invested on January 1st 2006 attracted an annual interest rate of 4%. By what factor does this sum increase each year and how much will it be worth on January 1st 2012. (25)

(b) Given that (2x-3) is a factor of $2x^3-x^2-7x+6$, find all 3 roots. (25)

(c) Solve for x:

(i) $\log_3(x+4)-\log_3 x=2$ (10)

(ii) $\ln\left(\frac{5x}{x-1}\right)=1.9$ (15)

(d) The spread of a virus through a city is modelled by the function (25)

$$N=\frac{15000}{1+100e^{-0.5t}}$$

where N is the number of people infected by the virus after t days. How many days will it take for 2000 people of this city to be infected with the virus?

4. (a) Find a and b if $a(3+4i)-b(1+2i)-5=0$ (20)

(b) Let z=(2+i). Show z is a solution of both:

(i) $z^2-4z+5=0$ (8)

(ii) $z^3-6z^2+13z-10=0$ (12)

(c) Express $\frac{(3+i)^2}{1-i}$ in the form a +ib. (20)

(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: 2i, -1+i , 2+i. (20)

(e) Express $\frac{2}{-1+i}$ in both a+ib and polar forms and find value of $\left(\frac{2}{-1+i}\right)^6$ (20)

5. (a) Express sin 3x +sin x as a product and hence find the values of x in the range $0 \leq x \leq 180°$ for which (sin x + sin 2x +sin 3x)=0. (25)

(b) Express both (sin 5x+sin x) and (cos 5x+ cos x) as products and hence solve: (25)

$$\frac{\sin 5x+\sin x}{\cos 5x+\cos x}=-1 \text{ in the range } 0 \leq x \leq 90^{o}$$

(c) In a triangle ABC, the lengths of the sides are: AB=17 cm; BC=9 cm; AC=15cm. Find the value of the internal angles. (25)

(d) (i) show that $\sin 3x \cos x-\cos 3x \sin x=\sin 2x$ (13)

(ii) $2\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)=\cos 2x$ (12)

6. C_1 is the circle $x^2+y^2+2x-2y-23=0$

C_2 is the circle $x^2+y^2-14x-2y+41=0$

(a) Prove that both circles touch externally and find the point of contact. (25)

(b) Find the centre and radius of the circle $x^2+y^2-6x-4y-36=0$ and determine if the points (5,4) ; (-1,3) and (-2, -7) are inside, on or outside the circle. (25)

(c) Find the equation of the line that passes through the point of intersection of the lines 3x+2y-1=0 and 2x-y+7=0 and is perpendicular to the line 2x-y+7=0. (25)

(d) Solve for x: $x-2=\sqrt{3x-2}$ (25)

7. (a) Evaluate 3 of the following integrals: (25 each)

(i) $\int_{3}^{5} \frac{2x+3}{(x-2)(x+5)} dx$

(ii) $\int_{2}^{5} \left(3x+5+\frac{1}{x^2} \right) dx$

(iii) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin 3x \sin x \, dx$

(iv) $\int_{0}^{3} \frac{x dx}{\sqrt{x^2+16}}$

(b) Sketch the curve $y=3x^2+1$ between x=-2 and x=4 calculate the area enclosed by the curve, the x axis and the lines x=-1 and x=3. (25)

8. (a) Find the inverse of the matrix $\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$ (20)

(b) Put the following set of linear equations into matrix form and hence solve the set: (20)

2x + 3y = 3
3x + 5y = 4

(c) If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ show that $A^2+I=2A$. (20)

(d) $A = \begin{pmatrix} 5 & 2 \\ -2 & 1 \end{pmatrix}$ and $X = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Find k so that $AX=kX$. (20)

(e) Find the 2 value which satisfy the matrix equation: (20)

$$\begin{pmatrix} 2 & k \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ k \end{pmatrix} = 20$$