

DUBLIN INSTITUTE OF TECHNOLOGY

Faculty of Engineering

First Year Entrance Examination 2009

In

MATHEMATICS

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

A table of formulae is attached

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1. (a) Solve the following equation for n: $\frac{n!}{(n-2)!} = 90$ (25)
- (b) Given that $x=-3$ is a root of the equation $x^3-x^2-17x-15=0$, find the other roots. (25)
- (c) A disc can be made to rotate at 2 different speeds. If it is made to rotate slower by 20 revs per hour, it would require 2.5 extra hours to make 100 revs. Find the higher speed at which it can operate. (50)
2. (a) A closed rectangular container with a square base is to have a volume of 2000 cm^3 . It costs twice as much per square cm for the top and the bottom as it does for the sides. Find the dimensions of the container associated with the least cost. (50)
- (b) A moving vehicle encounters a resistance to motion given by: $F = \frac{5}{x} + 100x$. Find the minimum resistance. (20)
- (c) Differentiate the following at the given points: (10 each)
- (i) $y = \sqrt{2x+1}$ at $x=1.5$
- (ii) $y = \frac{2+x}{2-x}$ at $x=1$
- (iii) $y = \cos 3x$ at $x = \frac{\pi}{2}$
3. (a) The 100 unit light source reduces by 12% for every 1 cm it passes through a sheet of glass. Set up a geometrical progression to describe this situation and give its first three terms. For what thickness of glass will the intensity of the light be halved and calculate by how much the intensity will be reduced by a sheet of thickness 10cm. (25)
- (b) Solve the equation $x - \sqrt{x+7} = 5$ (25)
- (c) Solve for x: (10)
- (i) $2 \ln(3x-5) = 4$
- (ii) $\ln\left(\frac{4+x}{3+2x}\right) = 0.5$ (15)
- (d) In a chemical reaction the amount of starting material P cc after t minutes is given by $P = 50e^{-0.5t}$. What is amount after 2 minutes? Estimate how long it will take for P to halve in value. (25)

4. (a) Find a and b if $(a+b) + i(a-b) = (2+i)^2 + i(4-3i)$ (20)
- (b) Express $\frac{-1+5i}{2+3i}$ in $a+ib$ form (20)
- (c) Find the 2 square roots of $-1+2i$ (20)
- (d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form; (20)
- $3i, 1-i, -1+2i$
- (e) Write the result of $\frac{1+3i}{1-2i}$ in polar form and hence evaluate z^{10} in the form $a+ib$. (20)
5. (a) An engineer has to make a triangular component from a metal sheet. (25)
The triangle is ABC and the length AB is 12 cm, the length BC is 9 cm while the included angle ABC is 115° . What are the other angles and the length of AC ?
- (b) From a window 8m above horizontal ground the angle of elevation of the top of a higher building across the road is 20° and, from the same window, the angle of depression of the foot of the same building is 30° . Find the width of the road and the height of the building. (25)
- (c) Sketch $y = \cos(2x)$ from $x=0$ to $x=180^\circ$ and mark the 2 points where $\cos(2x) = -0.5$ (25)
- (d) On the first day of use, a machine produces 600 items. One the next day production drops to 575 items and the amount produced continues to fall by the same amount each day.
- (i) How many items are produced on day 12? (5)
- (ii) What is total production for first 12 days? (10)
- (iii) On what day does production fall below 200 items? (10)

6. (a) Evaluate 3 of the following integrals (33 each)

(i) factorize $x^2 + 2x - 15$ and then evaluate $\int_2^7 \frac{3x+7}{x^2+2x-15} dx$

(ii) $\int_1^3 \left(x^2 + \frac{3}{x^2} - \sqrt{x} \right) dx$

(iii) $\int_0^{\frac{\pi}{3}} x \cos x dx$

(iv) $\int_{-1}^2 \frac{4x}{(4x^2+5)^3} dx$

7. (a) Find the inverse of the matrix: $\begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$ (25)

- (b) Put the following set of linear equations into matrix form and hence solve the set. (25)

$$7x + 2y = 12$$

$$3x + y = 5$$

- (c) Given $A = \begin{pmatrix} 3 & 2 \\ -3 & -4 \end{pmatrix}$ show $A^2 + A - 6I = 0$ (25)

- (d) $C = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ -1 & 0 & 3 \end{pmatrix}$: Find k so that $C^2 - 4C = kI$ (25)

8. (a) Find the centre and radius of the circle $x^2 + y^2 + 4x - 2y + 1 = 0$. (25)

- (b) Find the equation of the circle which touches the y axis at the point $(0,3)$ and passes through the point $(1,0)$. Hence find the co-ordinates of its centre and the length of its radius. (25)

- (c) The time for a journey is inversely proportional to the speed of the vehicle. Calculate the percentage increase or decrease in the journey time when the speed is (25)

(i) increased by 50%

(ii) decreased by 50%

(iii) increased from 50mph to 60 mph

(iv) decreased from 50 mph to 40 mph.

- (d) Solve for a : $2\log_3(2a^2+1)=10$ (25)