

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2016

In

MATHEMATICS

24th August 2016

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables and graph paper are available for use

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1. (a) Make x the subject of the formula: (25)

$$\frac{4y - 5x}{x} = 2y + \frac{3y}{2}$$

- (b) Factorize and find the roots of $z = y^2 - 2y - 3$ and $y = x^2 + 2x$. (25)

- (c) Simplify the following expression involving indices: (25)

$$x^2(y^2 + x^2y - 2xy) + \frac{x^2y^3}{y} + \frac{x^2y^3}{xy^2}$$

- (d) Expand the following binomial: $(2x - y)^4$. (25)

2. (a) Find values of the first derivatives of the following:

(i) $f(x) = \cos(2x^3 + 3x^2 - 5x)$ (25)

(ii) $g(x) = \frac{3x^2 - 2x}{x+2}$ (25)

- (b) Given the function $y = \frac{x^3}{3} - \frac{x^2}{2} - 6x + \frac{5}{3}$ Find the two turning points and specify if they are maximum or minimum points. (25)

- (c) The distance x metres moved by a car in a time t seconds is given by: (25)

$$x = 3t^3 - 2t^2 + 4t - 1$$

Determine the velocity and acceleration when $t = 1.5$ s.

3. (a) Solve for x :

(i) $\log_{10} (2x-1) - \log_{10} (x-1) = 1$ (15)

(ii) $\ln \left(\frac{x+3}{2-x} \right) = 1$ (15)

(b) In a chemical reaction, the amount of starting material in grams left after t hours is given by $M = 150e^{-0.1t}$.

What is the initial amount of M ? How much material is left after 4 hours and estimate how long it will take for M to fall to 5 grams. (30)

(c) The following table gives measurements of temperature T , in $^{\circ}\text{C}$, at various times t in minutes. The values of T are believed to be related to time t by the law $T = A e^{kt}$.

t in minutes	5	10	15	20	25
T $^{\circ}\text{C}$	150	406	1105	3003	8163

(i) Use the log-linear graph paper supplied to confirm this for the given table of values. (10)

(ii) Find the values of A and k . (30)

4. (a) Given $z = 2 + 3i$ and $h = 2 - 2i$. Calculate $z + h$, $z - h$, $z \cdot h$ and $\frac{z}{h}$. (25)

(b) Given $z = 2 + 2i$, calculate z^4, z^3, z^2 and express the results in rectangular and polar forms. (25)

(c) Find a if: $(2 + 2i)^2 + 6 - 2ai = 2a + 2i$. (25)

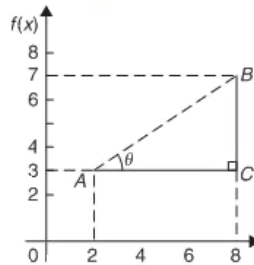
(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: $1 + 3i$, $3 - 1i$, $-3 - 1i$, $-1 + 3i$, $3i$ and $3 + 0i$. (25)

5. (a) Solve $\sin(2x) = 0.9$ for values of x between 0° and 360° . (25)

(b) The elevation of a tower from two points, one due East of the tower and one due West of it are 20° and 24° , respectively, and the two points of observation are 300 m apart. Find the height of the tower. (25)

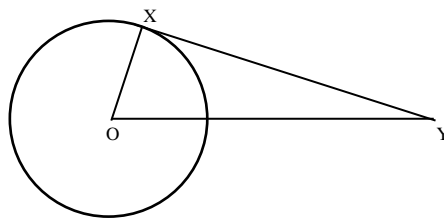
(c) In the following figure point A lies at coordinates (2,3) and point B at (8,7). Determine: (25)

- i) the distance AB.
- ii) the gradient of the straight line AB.
- iii) the angle AB makes with the horizontal (θ).



(d) Sketch the graphs for $y = \cos(x)$ and $h = \cos(2x)$. (25)

6. (a) In the crank mechanism shown below, XY is a tangent to the circle at point X . If the circle radius is 10 cm and length OY is 40 cm, determine the length of the connecting rod XY . (20)



(b) Find the area of a circle having a circumference of 60 mm. (20)

(c) Given the following values: (20)

x	0	10	20	30	40
y	5	35	65	95	125

Determine the slope, the x and y intercepts and the equation of the function.

(d) Find the equation of the line that passes through the point of intersection of the lines $2x + y + 2 = 0$ and $3x + y + 2 = 0$ and is perpendicular to the line $y - 3x = 3$. (40)

7. (a) Determine the following integrals:

$$(i) \int \left(\frac{5x^3 - 4x^2}{x} + 5 \right) dx \quad (25)$$

$$(ii) \int (2x + 3) \cos(x^2 + 3x) dx \quad (25)$$

$$(iii) \int (3e^{4x} + \cos(4x) + 5x^{-1}) dx \quad (25)$$

(b) Find the area under the curve $y = x^3 + 3x^2 - 3x + 10$ between the values $x = -2$ and $x = 2$. (25)

8. Given:

$$A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 6 & 3 \\ 2 & 3 & 3 \end{pmatrix}, C = \begin{pmatrix} 2 & 4 \\ 2 & 7 \end{pmatrix} \text{ and } D = \begin{pmatrix} 5 & 1 & 3 \\ 6 & 1 & 6 \\ 7 & 3 & 2 \end{pmatrix}$$

(a) Calculate the following: $A + C$, $A - C$ and $A \times C$. (30)

(b) Can the following calculations be performed? (30)

$$D \times A, C \times A, A \times B \text{ and } A \times D$$

Explain your reasoning for each and show the size of the result matrix.

(c) Solve the following system of simultaneous equations: (40)

$$\begin{aligned} x + 2y + 3z &= 3 \\ 2x + y + z &= 2 \\ 2x + 3y + z &= 1 \end{aligned}$$