DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2014

In

MATHEMATICS

August 2014

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables are available for use

Dr. Kevin Kelly

Dr Martin Rogers

Ms. Marisa Llorens-Salvador

1. (a) Make x the subject of the formula:

$$\frac{2x}{5} - \frac{3b}{4} = x + 5$$

(30)

(b) Factorize $x^2 + 8x + 15$ and $x^2 + x - 2$. Show the roots in a graph.

(30)

(c) Solve the simultaneous equations:

$$x + 3y + 2z = 2$$
$$2x + y + z = 6$$
$$x + y + z = 3$$

(40)

2. (a) Find values of the first derivatives of the following at the given points:

(25 each)

(i) $f(x) = e^{3t} \sin(3t)$ at t= 0

(ii) $g(x) = \sqrt{x^3 + 7x^2 + 3x + 4}$ at x=0

(b) Given the function y= $x^3 - 2x^2 - 7x - 5$. Find the turning points and specify if they are maximum or minimum points.

(25)

(c) Given the function $y = (x^2 + kx)e^x$; and that $\frac{dy}{dx} = 7 \ \ for$

$x = 0$, find the value of k.

(25)

3. (a) The temperature of a cooling liquid is measured at different times and the following results are obtained:

t (time)	10	15	20	25
T (Temperature)	152.67	168.73	186.48	206.09

Prove that the law relating time and temperature is of the form $T = A\, e^{kt}$, where A and k are constants. Use log-linear paper to support your calculations.
Determine the approximate value of A and k.

(40)

(b) Solve for x:

(i) $\log_{10}(x+3) + \log_{10}(x) = 2$ (15)

(ii) $\ln(\frac{x-2}{x+7}) = 4.2$ (15)

(c) The amount of chemical in a reaction after t seconds is given by

$M = 50\, e^{-0.1\, t}$ in grams.

How much material is left after 20 seconds and estimate how long it will take for the amount of the chemical to reduce to 5 grams?

(30)

4. (a) Given z= -2-4i and h= 5-5i. Calculate $z + h$, $z - h$, $z \cdot h$ and $\frac{z}{h}$.

(25)

(b) Show that $(2 - i)^3 - 2(1 + i)^2 - 2 + 15i = 0$.

(25)

(c) Express (-2+3i) in polar form and calculate $(-2+3i)^7$.

(25)

(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: -4i , 2+3i , 2-3i , -1-3i , 2+0i .

(25)

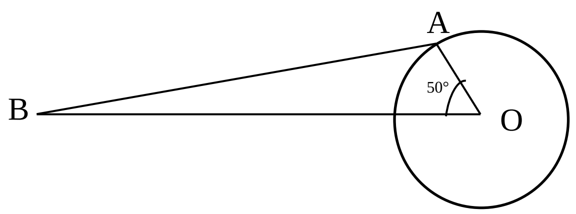
5. (a) A surveyor measures the angle of elevation of the top of a perpendicular building as 19°. He moves 120 m nearer the building and finds the angle of elevation is now 47°. Determine the height of the building.

(30)

(b) A crank mechanism of a petrol engine is shown in the figure below. Arm OA is 10 cm long and rotates clockwise about O. The connecting rod AB is 30 cm long and end B is constrained to move horizontally. Calculate:

a) The angle between the connecting rod AB and the horizontal and the length of OB for the given position.

b) How far does B move when angle AOB changes from 50° to 120°.



(35)

(c) Sketch the graph for 2cos(x) and cos(2x) between 0 and 2 π. Hence solve the following equation cos(2x)=-0.866 for $0 \leq x \leq 360°$

(35)

6 (a) A circle has centre (-1,1) and radius 3. Calculate the equation of the circle and where it cuts the x and y axis.

(30)

(b) If a car moves with constant acceleration a then its velocity is given by v=u+at, where u is the initial velocity and t is the time. Given the following measurement of velocities find the acceleration and initial velocity of the car. At time t=1 v=23 and at time t=10 v=50.

(35)

(c) Find the equation of the line that passes through the point of intersection of the lines x+y+1=0 and 2x-3y-8=0 and is parallel to the line 2y + x=2.

(35)

7 (a) Evaluate the following integrals: (25 each)

(i) $\int (\frac{2x^4}{x^2} + \frac{3}{x} + 8)\ dx$

(ii) $\int_0^{\pi/2} \cos(2x)\ \cos(5x)\, dx$

(iii) $\int (6x^2 + 4) \frac{1}{x^3+2x-8} dx$

(b) Find the area under the curve y = $x^3 - 5x + 15$ between the values x=0 and x=1.

(25)

8. (a) If A= $\begin{pmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{pmatrix}$ and B= $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ find value of k so that AB=kB

(30)

(b) Given the following matrices calculate: D*C, D*E, F*D , C*F and the determinant of D, if the operations are feasible. Explain the results.

$$C = \begin{pmatrix} 3 & 5 & 4 \\ 2 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix} \quad D = \begin{pmatrix} 5 & -3 \\ 1 & 5 \end{pmatrix} \quad E = \begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix} \quad F = \begin{pmatrix} 1 & 2 \\ 7 & 3 \\ 2 & 4 \end{pmatrix}$$

(35)

(c) Use Gaussian elimination to solve the following system of simultaneous equations:

$$3x + y = 1$$
$$2x - y = 5$$

(35)