

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2014

In

MATHEMATICS

April 2014

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables are available for use

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1 (a) Make v the subject of the formula:

$$E = \frac{1}{2}mv^2 + mgh$$

(30)

(b) Factorize $x^2 + 2x - 3$ and $x^2 - 4x - 21$. Show the roots in a graph.

(30)

(c) Solve the simultaneous equations:

$$\begin{aligned} 3x + 4y + z &= 18 \\ 4x + 5y + z &= 23 \\ x + y + z &= 3 \end{aligned}$$

(40)

2. (a) Find values of the first derivatives of the following at the given points:

(25 each)

(i) $f(x) = e^x \sin(2x)$ at $x=0$

(ii) $g(x) = \cos(x^3 + 2x)$ at $x=0$

(b) Given the function $y = x^3 - 7x^2 + x + 6$. Find the two turning points and specify if they are maximum or minimum points.

(25)

(c) Given the function $y = \frac{3x^2 + kx}{(x+1)}$ and that $\frac{dy}{dx} = 4$ for $x = 0$, find the value of k .

(25)

3. (a) The temperature of a cooling liquid is measured at different times and the following results are obtained:

T (Temperature)	92.2	55.9	33.9	20.6
t (time)	10	20	30	40

Prove that the law relating time and temperature is of the form $T = A e^{kt}$, where A and k are constants. Use log-linear paper to support your calculations. Determine the approximate value of A and k.

(40)

- (b) Solve for x:

(i) $\log_{10}(x+2) + \log_{10}(x-1) = 3$ (15)

(ii) $2\ln x - 2\ln(x+1) = 2$ (15)

- (c) The amount of chemical in a reaction after t seconds is given by

$$M = 500 e^{-0.05t} \text{ in grams.}$$

How much material is left after 120 seconds and estimate how long it will take for the amount of the chemical to reduce to 35 grams?

(30)

4. (a) Given $z = 2 - 2i$ and $h = 4 - i$. Calculate $z + h$, $z - h$, $z \cdot h$ and $\frac{z}{h}$.

(25)

- (b) Show that $z = 1 + i$ is a root of $z^3 + z^2 + z + 1 - 5i = 0$.

(25)

- (c) Express $(4 - 2i)$ in polar form and calculate $(4 - 2i)^4$.

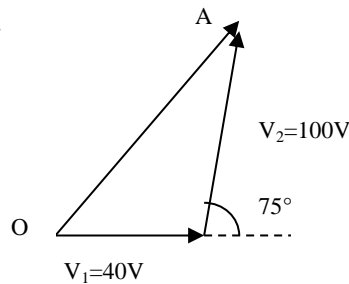
(25)

- (d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: $1 + i$, $-1 + i$, $-2 - 3i$, $-3i$, $5 - 5i$.

(25)

5. (a) The angle of depression of a ship viewed at a particular instant from the top of a 75 m vertical cliff is 30° . Find the distance of the ship from the base of the cliff at this instant. The ship is sailing away from the cliff at constant speed and 1 minute later its angle of depression from the top of the cliff is 20° . Determine the speed of the ship in km/h.
- (35)

- (b) Two voltage phasors are shown in the figure below. If $V_1 = 40\text{V}$ and $V_2 = 100\text{V}$, determine the value of their resultant (i.e. the length OA) and the angle the resultant makes with V_1 .



(20)

- (c) Sketch the graph for $3\sin(x)$ and $\sin(3x)$ between 0 and 2π . Hence solve the following equation $\sin(3x) = -0.5$ for $0 \leq x \leq 360^\circ$
- (45)

- 6 (a) Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at point $(4, -2)$. What are the centre and radius of the circle? Calculate the distance from the centre to $(4, -2)$.
- (30)

- (b) The line segment joining the points $(4, 1)$ and $(8, 17)$ is parallel to the line segment joining the points $(-2, 5)$ and $(0, a)$. What is the value of a ?
- (30)

- (c) Find the equation of the line that passes through the point of intersection of the lines $3x + y - 1 = 0$ and $x - 2y - 5 = 0$ and is perpendicular to the line $y + x = 3$.
- (40)

7 (a) Evaluate the following integrals: (25 each)

(i) $\int 8 - 5x - \frac{7x^3}{x^4} dx$

(ii) $\int (2x + 3) \frac{1}{x^2 + 3x + 8} dx$

(iii) $\int \frac{5x-2}{(x-1)(x+2)} dx$

(b) Find the area under the curve $y = x^2 + 6x + 2$ between the values $x=0$ and $x=3$. (25)

8. (a) Find k:

$$\begin{pmatrix} 3 & 7 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 11 \\ 11 & 10 \end{pmatrix}$$

(30)

(b) Given the following matrices calculate: $B \cdot C$, $D \cdot C$, $B \cdot D$, $D \cdot B$ and the determinant of D , if the operations are feasible. Explain the results.

$$B = \begin{pmatrix} 3 & 4 & 4 \\ 1 & 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 3 \\ 2 & -3 \\ 1 & 3 \end{pmatrix} \quad D = \begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix} \quad E = \begin{pmatrix} 5 & 1 & 2 \\ 9 & 7 & 3 \\ 10 & 2 & 4 \end{pmatrix}$$

(35)

(c) Use Gaussian elimination to solve the following system of simultaneous equations:

$$\begin{aligned} x + 3y &= 7 \\ 2x + 3y &= 3 \end{aligned}$$

(35)