

**DUBLIN INSTITUTE OF TECHNOLOGY**

**First Year Engineering Entrance Examination 2015**

**in**

**MATHEMATICS**

**August 2015**

**Dr. Kevin Kelly**

**Mr. Kevin Gaughan**

**Ms. Marisa Llorens-Salvador**

Attempt any **6** of the following 8 questions

Time Allowed: 3 hours

Each question has 100 marks

All questions carry equal marks

Maths Tables and graph paper are available for use

1. (a) Make  $x$  the subject of the formula: (30)

$$y = \frac{(x + 4)}{a} + 5b$$

- (b) Factorize  $y = x^2 + x - 2$  and  $z = x^2 + 8x + 15$ . Show the roots on a graph. (20)

- (c) Simplify the following expression involving indices: (20)

$$\frac{x^2y^2 + x^5y - x^4y^3}{x^2y^2}$$

- (d) Solve the simultaneous equations: (30)

$$\begin{aligned}x + 2y + z &= 1 \\x + 3y + z &= 1 \\4x + 2y + z &= 5\end{aligned}$$

2. (a) Find values of the first derivatives of the following at the given points:

(i)  $f(x) = \frac{\sin(3x)}{\cos(2x)}$  at  $x = 0$  (25)

(ii)  $g(x) = \sin(5x^3 + 2x^2 + 2x)$  at  $x = 0$  (25)

- (b) Given the function  $y = 3x^3 + 2x^2 + 5$ . Find the two turning points and specify if they are maximum or minimum points. (25)

- (c) Given the function  $y = \frac{3x}{k(\cos(2x))}$  and that  $\frac{dy}{dx} = 3$  for  $x = 0$ , find the value of the constant  $k$ . (25)

3. (a) Solve for  $x$ :

(i)  $\log(x + 3) - \log(x) = 2$  (15)

(ii)  $\ln\left(\frac{x-1}{x+5}\right) = 5.2$  (10)

(b) In a chemical reaction, the amount of starting material in  $\text{cm}^3$  left after  $t$  hours is given by: (25)

$$M = 24e^{-0.2t}$$

(i) What is the initial amount of  $M$ ?

(ii) How much material is left after 3 hours?

(iii) How long it will take for  $M$  to fall to  $1 \text{ cm}^3$ ?

(c) The following table gives measurements of temperature  $T$ , in  $^\circ\text{C}$ , at various times  $t$  minutes. The values of  $T$  are believed to be related to time  $t$  by the law  $T = A e^{kt}$ .

$t$ in minutes	10	15	20	25	30
$T$ $^\circ\text{C}$	247.31	317.55	407.74	523.55	672.25

(i) Use the log-linear graph paper supplied to confirm this for the given table of values. (20)

(ii) Find the values of  $A$  and  $k$ . (30)

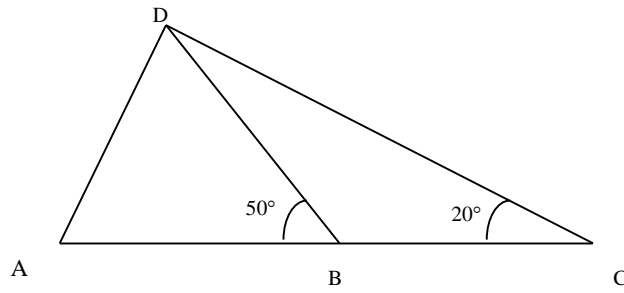
4. (a) Given  $z = -1 - 3i$  and  $h = 3 + 2i$ . Calculate  $z + h$ ,  $z - h$ ,  $z \cdot h$  and  $\frac{z}{h}$ . (25)

(b) Show that  $(2 - i)^3 - 2(1 + i)^2 - 2 + 15i = 0$ . (25)

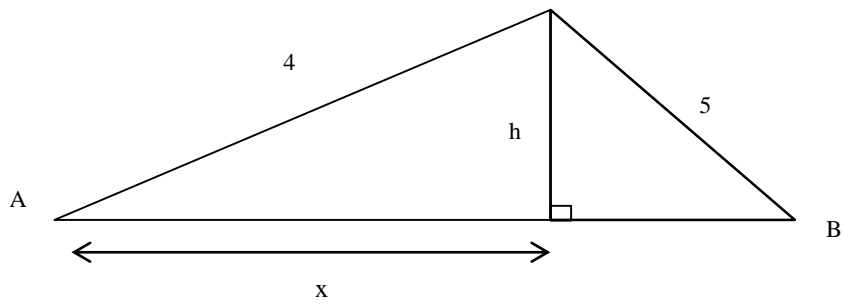
(c) Express  $(-1 + 3i)$  in polar form and calculate  $(-1 + 3i)^4$ . (25)

(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form:  $2i$ ,  $2 + 2i$ ,  $2 - 2i$ ,  $-2 - 2i$ ,  $-2 + 2i$ ,  $2 + 0i$ . (25)

5. (a) In the diagram  $BC = 15$  m and  $AD = 8.4$  m . Find  $DC$ ,  $DB$  and the angle  $DAB$ . (30)



- (b) In the triangle below, the distance AB is 7 cm. Find  $x$ . (35)



- (c) Sketch the graphs of  $\sin(x)$  and  $\sin(2x)$  between  $0$  and  $2\pi$ . Hence, solve the following equation  $\sin(2x) = 0.866025$  for values of  $x$  such that  $0 \leq x \leq 180^\circ$ . (35)

6. (a) The points  $A(2,1)$  and  $B(8,9)$  are at the ends of a diameter of a circle.
- (i) Find the equation of the circle. (10)
- (ii) Draw a sketch of the circle and the points  $A$  and  $B$ . Show that  $AB$  is a diameter of the circle. (20)

- (b) The following values of temperature are obtained from a test:

$t$ time (minutes)	0	1	2	3	4
$T$ Temperature $^{\circ}\text{C}$	5	7	9	11	13

- (i) Plot a graph with  $T$  in the vertical axes and  $t$  in the horizontal axes. (10)
- (ii) Determine the gradient of the graph, the  $x$  axis intercept and the equation of the graph. (15)
- (iii) What is the value of  $x$  when  $T = 25^{\circ}\text{C}$ ? (5)
- (c) Find the equation of the line that passes through the point of intersection of the lines  $x - 4y + 5 = 0$  and  $2x + y - 2 = 0$  and is perpendicular to the line  $2y + x = 1$ . (40)

7. (a) Determine the following integrals:

(i)  $\int \left( x^4 + 4x^{\frac{5}{2}} + \frac{1}{x} + \frac{1}{x^3} \right) dx$  (25)

(ii)  $\int_0^{\pi} \sin(2x) \cos(x) dx$  (25)

(iii)  $\int (3x^2 + 2)\sqrt{(x^3 + 2x - 12)} dx$  (25)

- (b) Find the area under the curve  $y = x^3 + 2x^2 + 7x + 20$  between the values  $x = 0$  and  $x = 2$ . (25)

8. Given:

$$A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 10 \\ 8 & 7 \end{pmatrix}, C = \begin{pmatrix} 4 & 3 \\ 1 & 1 \\ 7 & 6 \end{pmatrix} \text{ and } D = \begin{pmatrix} 5 & 6 & 7 \\ 4 & 3 & 8 \end{pmatrix}$$

(a) Calculate the following:  $A + B$ ,  $A - B$  and  $A \times B$ . (25)

(b) Can the following calculations be performed:  $B \times A$ ,  $C \times A$ ,  $A \times C$  and  $A \times D$ ? (25)  
Explain your reasoning for each.

(c) Find the inverse of  $B$ . (20)

(d) Given the following determinant, find the constant  $k$ . (30)

$$\begin{vmatrix} 1 & k & k+1 \\ 3 & 2 & 1 \\ 2 & 2 & -2 \end{vmatrix} = 16$$